

Journal #20
Scope & Applications

If mathematics is
'out there' in the world,
then where exactly can
it be found?

Journal #20 CR
Scope & Applications

How does
mathematics
create our
scientific
reality?

Journal #21
Scope & Applications

Is mathematics
discovered or
invented?

Journal #21 CR
Scope & Applications

Is there a
distinction
between truth
and certainty in
mathematics?

Journal #22
Concepts & Language

What are
examples of
important terms
used in math?
How are symbols
utilized in math?

| Term | Correct definition | Example |
|------------------------|---|--|
| Definition* | A proposition of the form "All A is B and all B is A." | "A quadrilateral with four right angles is a rectangle." |
| Postulate | Statement accepted without proof. | "Through any two points there is exactly one line." |
| Axiom* | Same as postulate. | "Things which are equal to the same thing are equal to each other." |
| Conjecture* | An unproven statement that seems to be true. Sources of inspiration may vary. | Goldbach's Conjecture: Every integer greater than 4 can be expressed as a sum of primes. |
| Theorem* | A proven mathematical statement that logically follows from axioms, postulates, definitions, or previously proven theorems. | "If two intersecting lines form one right angle, then they form four right angles." |
| Proof** | A valid sequence of deductive steps, using accepted mathematical truths to arrive at a novel conclusion. | |
| Corollary to a theorem | A statement whose proof directly follows from the theorem. | "All right angles are congruent." |

**Key
Math
Terms**

Journal #22 CR
Concepts & Language

Can mathematics be characterized as a universal language?

Journal #23
Methodology

What are the steps used in the 'mathematical method' for creating knowledge in math?

Introduction to Proofs

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A proof : is a valid argument that establishes the truth of a mathematical statement.

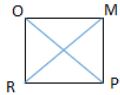
There are two types of proofs :

- **Formal proof** : where all steps are supplied and the rules for each step in the argument are given
- **Informal proof** : where more than one rule of inference may be used in each step, where steps may be skipped, where the axioms being assumed and rule of inference used are not explicitly stated.

Two-Column Geometric Proofs

| Statements | Reasons |
|--|---|
| The items we include in this portion of our two-column geometric proof will show the progression of our argument. They are the claims we believe to be true. | The items we include in this portion of our two-column geometric proof will explain why the corresponding statements are true. They justify any claims we make. |

Given: ROMP is a square
 Prove: $\overline{RM} \cong \overline{PO}$



| Statement | Reasons |
|--|--------------------------------|
| 1. ROMP is a square | 1. Given |
| 2. $\overline{RO} \cong \overline{MP}$ | 2. Definition of square |
| 3. $\overline{RP} \cong \overline{MO}$ | 3. Definition of square |
| 4. $\angle R \cong \angle M$ | 4. Definition of parallelogram |
| 5. $\triangle ORP \cong \triangle PMO$ | 5. SAS Congruence |
| 6. $\overline{RM} \cong \overline{PO}$ | 6. CPCTC |

Two Column Geometric Sample Proof Methodology

PROOF 1

Sample Proof Using Mathematical Induction

Brandon Grasley
 2014-01-21

Theorem
 For any $n \in \mathbb{N}$,

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

Proof
 Base case $n = 1$: If $n = 1$, the left side is 1 and the right side is $\frac{1(1)}{2} = 1$. So, the theorem holds when $n = 1$.
 Inductive hypothesis: Suppose the theorem holds for all values of n up to some $k, k \geq 1$.
 Inductive step: Let $n = k + 1$. Then our left side is

$$\begin{aligned} \sum_{i=1}^{k+1} i &= (k+1) + \sum_{i=1}^k i \\ &= (k+1) + \frac{k(k+1)}{2}, \text{ by our inductive hypothesis} \\ &= \frac{2(k+1)}{2} + \frac{k(k+1)}{2} \\ &= \frac{2(k+1) + k(k+1)}{2} \\ &= \frac{(k+1)(k+2)}{2} \end{aligned}$$

which is our right side. So, the theorem holds for $n = k + 1$. By the principle of mathematical induction, the theorem holds for all $n \in \mathbb{N}$.

Sample Proof Methodology

Journal #23 CR
Methodology

How does 'mathematical proof' differ from 'good reasons' for belief in other areas of knowledge?

Journal #24
Historical Development

How has math progressed and developed over time? Who makes up the mathematical community?

| | | |
|--|--|--|
| Measure/Shape (77,000 BC - 600 AD) | Math-Language (600-1600 AD) | Modern Math (1600-today) |
| <ul style="list-style-type: none"> 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 "Hindu Numerals" "Arithmetica" "Almagest" Trig-Tables Geometry Theory Shape Theory Egypt/Sumer Math Egypt/Sumer Count geometric design | <ul style="list-style-type: none"> logarithms Napier Euler's "Elements" power series "Liber Abaci" roots exponents Algebra (Al-Jabr) 0, roots, trig, series integrals, Sturians | <ul style="list-style-type: none"> log(probability) Information Theory Chaos, Set Theory, Stat Boolean Algebra Groups, N-D Geometry Bayes Theorem Graph Theory Calculus, Linear Algebra Probability Theory Analytic Geometry |

A Math Timeline
Historical Development

Journal #25
Links to Personal Knowledge & Scope

Do the terms 'beauty' or 'elegance' have a role in mathematical thought? How might math have value beyond the numbers?

ψ

$\psi(x) = \sum_{n \leq x} \Lambda(n)$

$\psi(x) \sim x$

$\psi(x) - x = O(\sqrt{x})$

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$\psi(x) - x = O(x^{1/3})$

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$\psi(x) - x = O(x^{1/198})$

$\psi(x) - x = O(x^{1/199})$

$\psi(x) - x = O(x^{1/200})$

$1 \times 1 = 1$

$11 \times 11 = 121$

$111 \times 111 = 12321$

$1111 \times 1111 = 1234321$

$11111 \times 11111 = 123454321$

$111111 \times 111111 = 12345654321$

$1111111 \times 1111111 = 1234567654321$

$11111111 \times 11111111 = 123456787654321$

$111111111 \times 111111111 = 12345678987654321$

#25 CR Prompt
Connections/Comparisons

How is the knowledge that math offers different from other areas of knowledge and why?

Knowledge Framework

Connections/Comparisons

| | |
|---------------------------|---|
| <p>Scope/applications</p> | <ul style="list-style-type: none"> • mathematics concerned with quantity, shape, space and change—difficult to define • used to create models in the natural and human sciences • the possibility of a mathematical treatment is taken by many to be the sign of intellectual rigour—for example, in economics or psychology • possesses qualities such as beauty and elegance—sometimes thought of as an art form • seems to be broadly universal and not tied to a particular culture • mathematical truths seem to be certain and timeless |
| <p>Concepts/language</p> | <ul style="list-style-type: none"> • uses a precisely defined set of symbols standing for abstract things like sets and relations • key terms such as axiom, deduction rule, conjecture, theorem, proof |

Knowledge Framework

Connections/Comparisons

| | |
|------------------------------------|--|
| <p>Methodology</p> | <ul style="list-style-type: none"> • uses pure reason from axioms to produce proofs of mathematical theorems • a statement in mathematics is true if and only if it is proved • mathematics does not seem to rely on sense perception of the world • mathematicians require intuition and imagination in order to prove theorems |
| <p>Historical development</p> | <ul style="list-style-type: none"> • seminal developments such as negative or irrational numbers have led to big changes in the way we view the world • numbers and geometry particularly important in historical development of other fields such as painting, architecture and music |
| <p>Links to personal knowledge</p> | <ul style="list-style-type: none"> • maths ability often taken to be a proxy for intelligence with consequences for individual self-esteem • much scope for major contributions to mathematics by talented individuals who cannot always explain the source of their insights, often ascribing them to intuition, imagination or emotion |