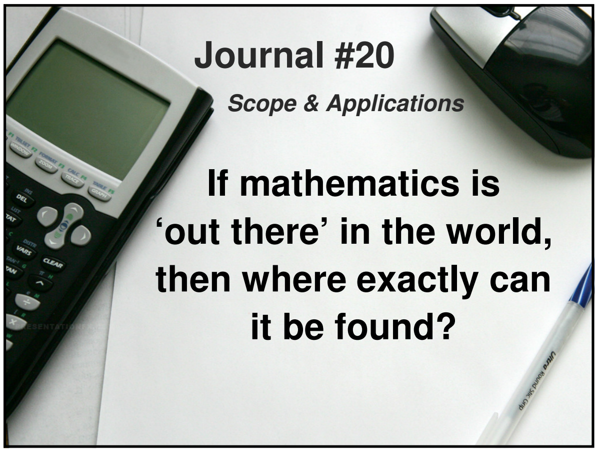
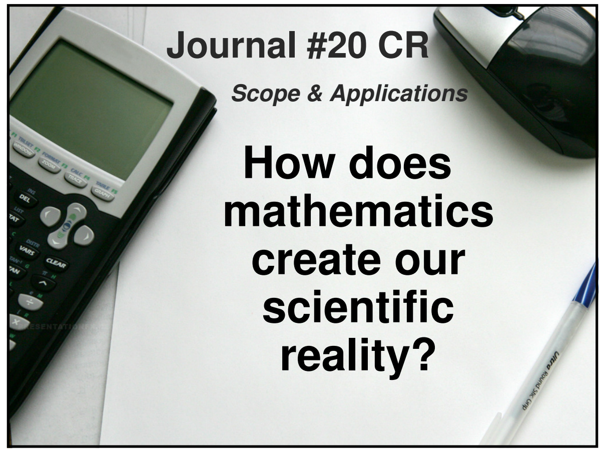
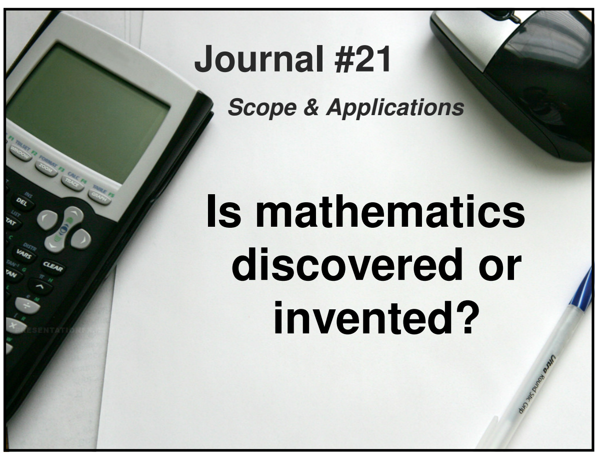
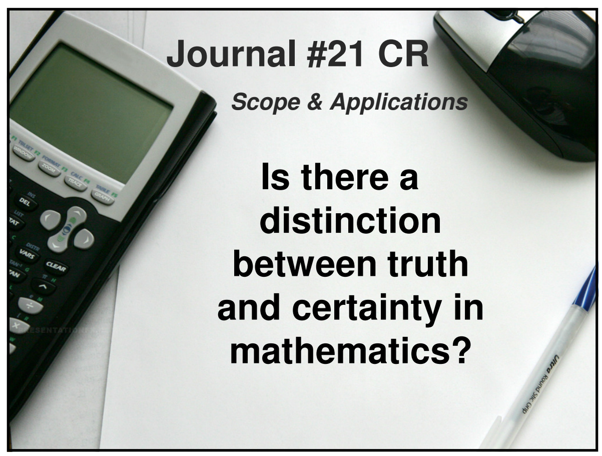
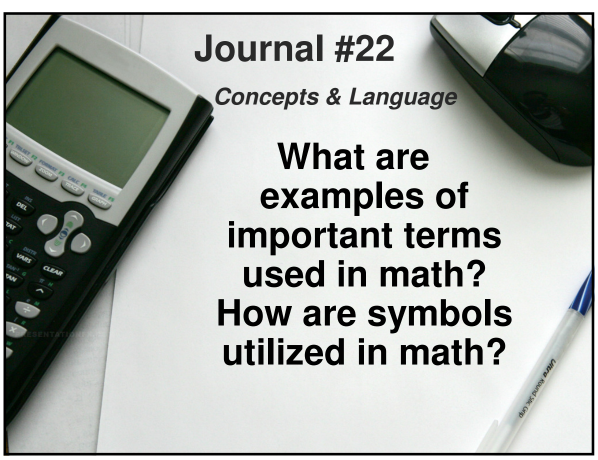
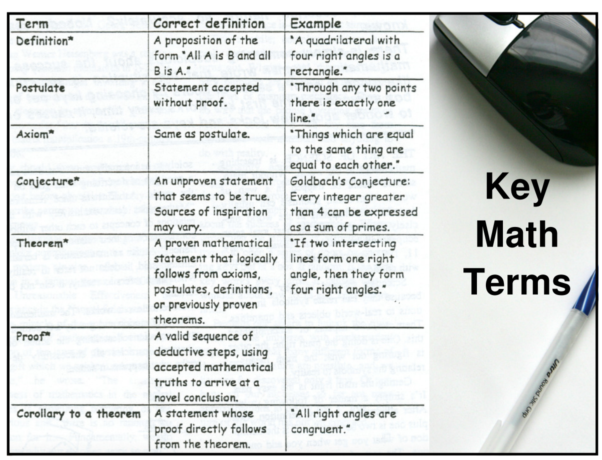












Term	Correct definition	Example
Definition*	A proposition of the form "All A is B and all B is A."	"A quadrilateral with four right angles is a rectangle."
Postulate	Statement accepted without proof.	"Through any two points there is exactly one line."
Axiom*	Same as postulate.	"Things which are equal to the same thing are equal to each other."
Conjecture*	An unproven statement that seems to be true. Sources of inspiration may vary.	Goldbach's Conjecture: Every integer greater than 4 can be expressed as a sum of primes.
Theorem*	A proven mathematical statement that logically follows from axioms, postulates, definitions, or previously proven theorems.	"If two intersecting lines form one right angle, then they form four right angles."
Proof*	A valid sequence of deductive steps, using accepted mathematical truths to arrive at a novel conclusion.	
Corollary to a theorem	A statement whose proof directly follows from the theorem.	"All right angles are congruent."

Journal #22 CR

Concepts & Language

Can mathematics be characterized as a universal language?

Journal #23

Methodology

What are the steps used in the 'mathematical method' for creating knowledge in math?

Introduction to Proofs

3

A proof : is a valid argument that establishes the truth of a mathematical statement.

There are two types of proofs :

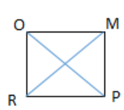
- Formal proof : where all steps are supplied and the rules for each step in the argument are given
- Informal proof : where more than one rule of inference may be used in each step, where steps may be skipped, where the axioms being assumed and rule of inference used are not explicitly stated.

Two-Column Geometric Proofs

Statements	Reasons
The items we include in this portion of our two-column geometric proof will show the progression of our argument. They are the claims we believe to be true.	The items we include in this portion of our two-column geometric proof will explain why the corresponding statements are true. They justify any claims we make.

Given: ROMP is a square

Prove: $\overline{RM} \cong \overline{PO}$



Statement	Reasons
1. ROMP is a square	1. Given
2. $\overline{RO} \cong \overline{MP}$	2. Definition of square
3. $\overline{RP} \cong \overline{MO}$	3. Definition of square
4. $< R \cong < M$	4. Definition of parallelogram
5. $\Delta ORP \cong \Delta PMO$	5. SAS Congruence
6. $\overline{RM} \cong \overline{PO}$	6. CPCTC

PROOF 1

Two Column Geometric Sample Proof

Methodology

Sample Proof Using Mathematical Induction

Brandon Grasley

2014-01-21

Theorem

For any $n \in \mathbb{N}$,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Proof

Base case $n = 1$: If $n = 1$, the left side is 1 and the right side is $\frac{1(2)}{2} = 1$.

So, the theorem holds when $n = 1$.

Inductive hypothesis: Suppose the theorem holds for all values of n up to some k, $k \geq 1$.

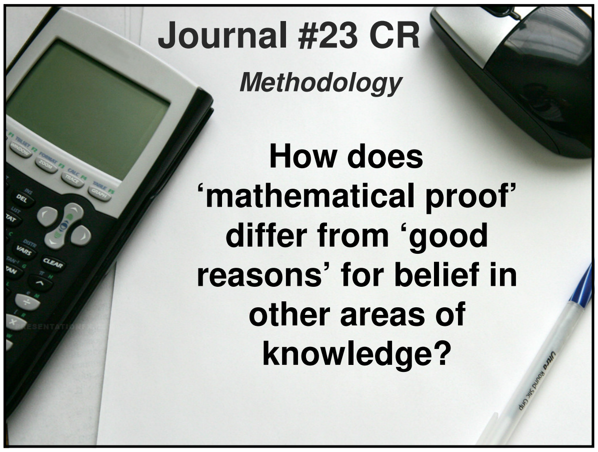
Inductive step: Let $n = k + 1$. Then our left side is

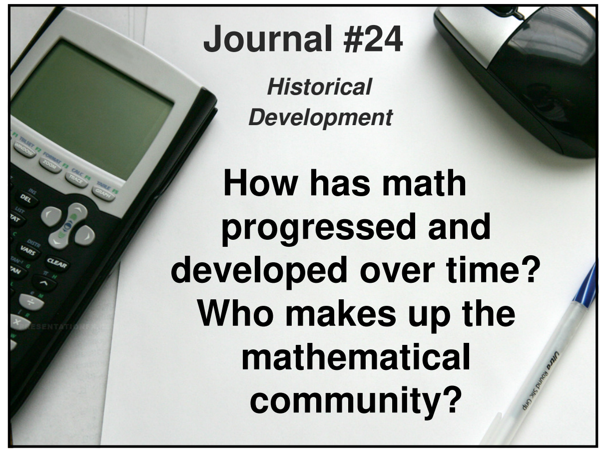
$$\begin{aligned}\sum_{i=1}^{k+1} i &= (k+1) + \sum_{i=1}^{k} i \\ &= (k+1) + \frac{k(k+1)}{2}, \text{ by our inductive hypothesis} \\ &= \frac{2(k+1)}{2} + \frac{k(k+1)}{2} \\ &= \frac{2(k+1) + k(k+1)}{2} \\ &= \frac{(k+1)(k+2)}{2}\end{aligned}$$

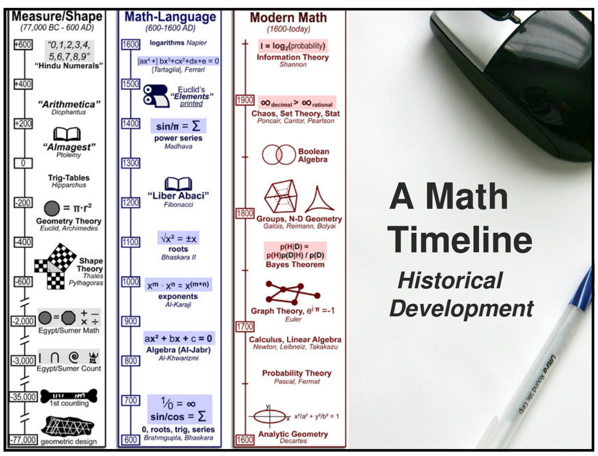
which is our right side. So, the theorem holds for $n = k + 1$. By the principle of mathematical induction, the theorem holds for all $n \in \mathbb{N}$.

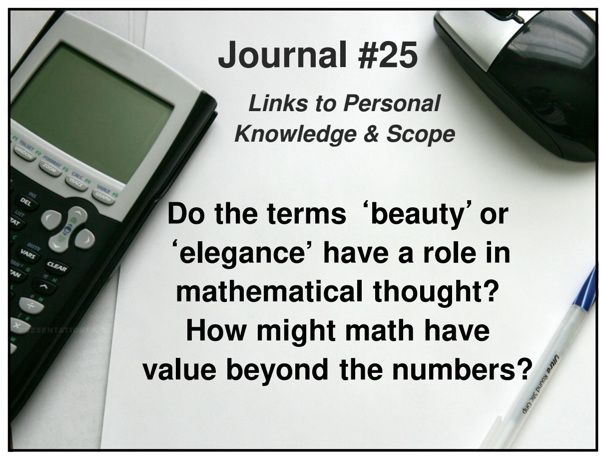
Sample Proof

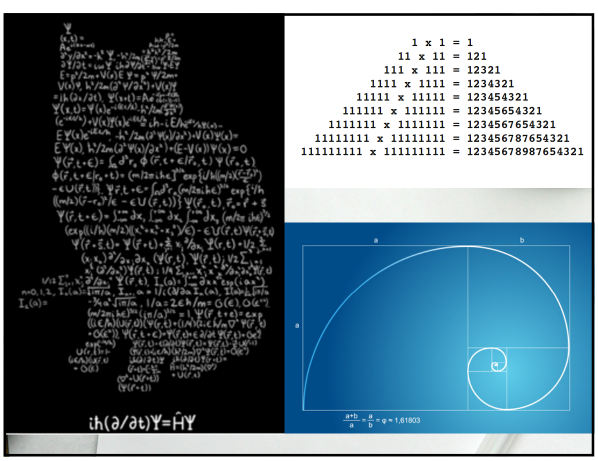
Methodology

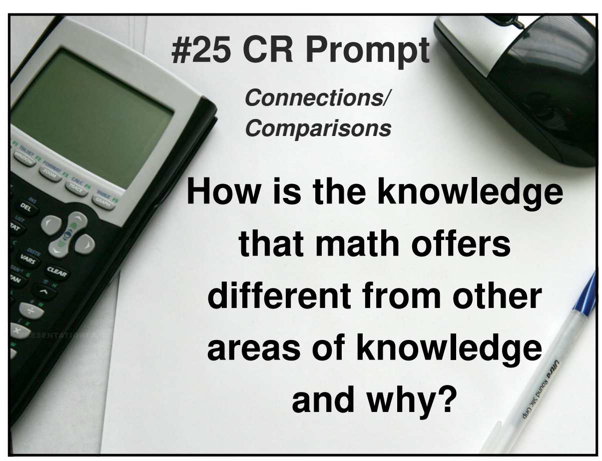

Journal #23 CR
Methodology
How does 'mathematical proof' differ from 'good reasons' for belief in other areas of knowledge?

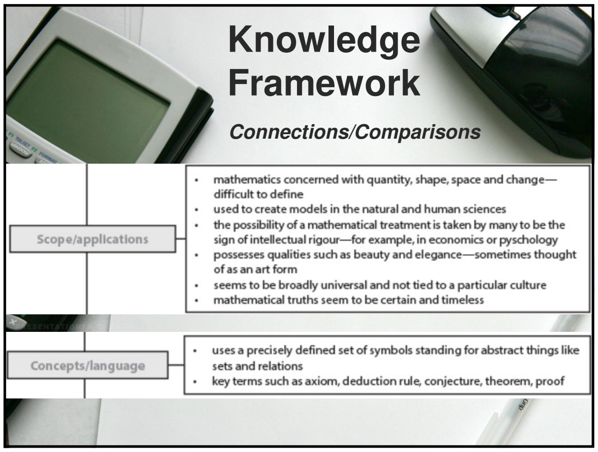

Journal #24
Historical Development
How has math progressed and developed over time? Who makes up the mathematical community?

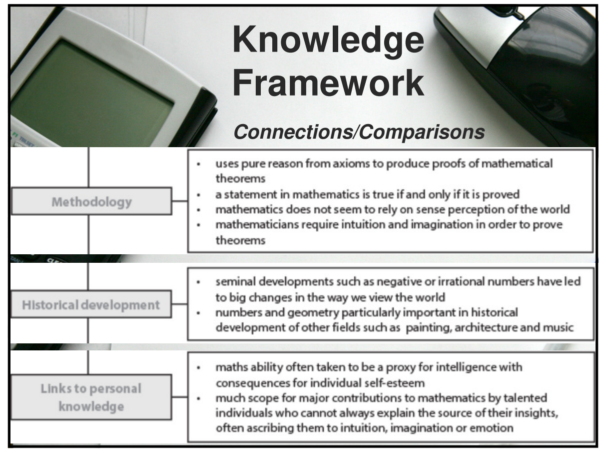

Measure/Shape
(77,000 BC - 600 AD)
Math-Language
(600-1600 AD)
Modern Math
(1600-today)
A Math Timeline
Historical Development


Journal #25
Links to Personal Knowledge & Scope
Do the terms 'beauty' or 'elegance' have a role in mathematical thought? How might math have value beyond the numbers?


iħ(∂/∂t)Ψ=ĤΨ
1 x 1 = 1
11 x 11 = 121
111 x 111 = 12321
1111 x 1111 = 1234321
11111 x 11111 = 123454321
111111 x 111111 = 12345654321
1111111 x 1111111 = 1234567654321
11111111 x 11111111 = 123456787654321
111111111 x 111111111 = 12345678987654321


#25 CR Prompt
Connections/ Comparisons
How is the knowledge that math offers different from other areas of knowledge and why?


Knowledge Framework
Connections/Comparisons
Scope/applications
• mathematics concerned with quantity, shape, space and change—difficult to define
• used to create models in the natural and human sciences
• the possibility of a mathematical treatment is taken by many to be the sign of intellectual rigour—for example, in economics or pyschology
• possesses qualities such as beauty and elegance—sometimes thought of as an art form
• seems to be broadly universal and not tied to a particular culture
• mathematical truths seem to be certain and timeless
Concepts/language
• uses a precisely defined set of symbols standing for abstract things like sets and relations
• key terms such as axiom, deduction rule, conjecture, theorem, proof


Knowledge Framework
Connections/Comparisons
Methodology
• uses pure reason from axioms to produce proofs of mathematical theorems
• a statement in mathematics is true if and only if it is proved
• mathematics does not seem to rely on sense perception of the world
• mathematicians require intuition and imagination in order to prove theorems
Historical development
• seminal developments such as negative or irrational numbers have led to big changes in the way we view the world
• numbers and geometry particularly important in historical development of other fields such as painting, architecture and music
Links to personal knowledge
• maths ability often taken to be a proxy for intelligence with consequences for individual self-esteem
• much scope for major contributions to mathematics by talented individuals who cannot always explain the source of their insights, often ascribing them to intuition, imagination or emotion